

Top ten accelerating cosmological models

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Recent astronomical observations indicate that the Universe is presently almost flat and undergoing a period of accelerated expansion. Basing on Einstein's general relativity all these observations can be explained by the hypothesis of a dark energy component in addition to cold dark matter (CDM). Because the nature of this dark energy is unknown, it was proposed some alternative scenario to explain the current accelerating Universe. The key point of this scenario is to modify the standard FRW equation instead of mysterious dark energy component. The standard approach to constrain model parameters, based on the likelihood method, gives a best-fit model and confidence ranges for those parameters. We always arbitrary choose the set of parameters which define a model which we compare with observational data. Because in the generic case, the introducing of new parameters improves a fit to the data set, there appears the problem of elimination of model parameters which can play an insufficient role. The Bayesian information criteria of model selection (the AIC and BIC) are dedicated to promotion a set of parameters which should be incorporated to the model. We divide class of all accelerating cosmological models into two groups according to the two types of explanation acceleration of the Universe. Then the Bayesian framework of model selection is used to determine the set of parameters which gives preferred fit to the SNIa data. We find a few of flat cosmological models which can be recommend by both the Bayes factor and Akaike information criterion.

I. INTRODUCTION

Recent measurements of distant supernovae (SNIa) [1, 2] as well as current measurements of cosmic microwave background anisotropies [3] favor a spatially flat Universe filled by cold dark matter (CDM) [4, 5] and a dark energy component of unknown origin [1, 2], which can be modelled as a perfect fluid with energy density ρ_X and pressure p_X violating the strong energy condition $\rho_X + 3p_X > 0$. The combination of CMB and SNIa data with other orthogonal measurements, as the HST determination of the Hubble parameter, constrain the Universe to be almost flat even if we consider the time variation of dark energy equation of state [6]. All these models explaining current acceleration of the Universe in terms of smoothly distributed and slowly varying 'dark energy' are formulated in the context of the standard cosmological picture based on general relativity theory. Possible types of explanation include a cosmological constant Λ [7], an evolving scalar field (quintessence) [8], the phantom energy [9, 10] in which a weak energy condition is violated, models filled with Chaplygin gas [11], models with dynamical coefficient equation of state $w_X \equiv p_X/\rho_X$ (decaying vacuum energy density), usually parameterized by the scale factor a or redshift z , where $(1+z = a^{-1})$, fluid which describe quantum effects coming from massless scalar field (the Casimir effect) [12, 13, 14], the noninteracting Chaplygin gas and baryonic matter [15, 16, 17]. In Table I we collected the ten candidates for dark energy description together with their Friedmann first integral

$$3H^2 = \rho_{\text{eff}} - \frac{3k}{a^2}, \quad (1)$$

where $H = (d \ln a)/(dt)$ is the Hubble function, and $k = 0, \pm 1$ is the curvature index). For all these approaches some hypothetical dark energy component is postulated which satisfies the conservation condition

$$\dot{\rho}_i = -3H(\rho_i + p_i), \quad i = m, X \quad (2)$$

for both standard dust matter as well as dark energy X separately.

On the other hand, alternative ideas to the dark energy idea have been recently offered. Freese and Lewis [18] proposed so called the Cardassian model, in which the Universe is flat, matter dominated and accelerating not due to dark energy but as a consequence of modification of the Friedmann first integral as follows

$$3H^2 = \rho + B\rho^n, \quad (3)$$

TABLE I: The Hubble function for 10 prototypes of cosmological models explaining the present acceleration of the Universe in terms of dark energy

case model	$H^2(z)$ relation	independent model parameters
1 Λ CDM model	$H^2 = H_0^2 \{\Omega_{m,0}(1+z)^3 + (1 - \Omega_{m,0})\}$	$H_0, \Omega_{m,0}$
2 non-flat FRW model with Λ	$H^2 = H_0^2 \{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\Lambda,0}\}$	$H_0, \Omega_{m,0}, \Omega_{\Lambda,0}$
3 FRW model with 2D topological defects $p_X = -\frac{2}{3}\rho_X$	$H^2 = H_0^2 \{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\text{topo}}(1+z)\}$	$H_0, \Omega_{m,0}, \Omega_{k,0}$
4 FRW model with phantom dark energy $p_X = -\frac{4}{3}\rho_X$	$H^2 = H_0^2 \{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{ph,0}(1+z)^{-1}\}$	$H_0, \Omega_{m,0}, \Omega_{k,0}$
5 FRW model with phantom dark energy $p_X = w_X \rho_X$, $w_X < -1$ fitted	$H^2 = H_0^2 \{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{ph,0}(1+z)^{3(1+w_X)}\}$	$H_0, \Omega_{m,0}, \Omega_{k,0}, w_X$
6 FRW model with Chaplygin gas $p_X = -\frac{A}{\rho_X}$, $A > 0$	$H^2 = H_0^2 \left\{ \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{Ch,0} [A_S + (1 - A_S)(1+z)^6]^{\frac{1}{2}} \right\}$	$H_0, \Omega_{m,0}, \Omega_{k,0}, A_S$
7 FRW model with generalized Chaplygin gas $p_X = -\frac{A}{\rho_X^\alpha}$, $A > 0$, $\alpha = \text{const}$	$H^2 = H_0^2 \left\{ \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{Ch,0} [A_S + (1 - A_S)(1+z)^{3(1+\alpha)}]^{\frac{1}{1+\alpha}} \right\}$	$H_0, \Omega_{m,0}, \Omega_{k,0}, A_S, \alpha$
8a FRW models with dynamical E.Q.S. parameterized by z $p_X = (w_0 + w_1 z)\rho_X$	$H^2 = H_0^2 \left\{ \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{X,0}(1+z)^{3(w_0 - w_1 + 1)} \exp[3w_1 z] \right\}$	$H_0, \Omega_{m,0}, \Omega_{X,0}, w_0, w_1$
8b FRW models with dynamical E.Q.S parameterized by scale factor a $p_X = (w_0 + w_1(1-a))\rho_X$	$H^2 = H_0^2 \left\{ \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{X,0}(1+z)^{3(w_0 + w_1 + 1)} \exp\left[-\frac{3w_1 z}{1+z}\right] \right\}$	$H_0, \Omega_{m,0}, \Omega_{X,0}, w_0, w_1$
9 FRW model with quantum effects origin from massless scalar field at low temperature (Casimir effect) $\rho_X = -\frac{\rho_{X,0}}{a^4}$, $\rho_{X,0} > 0$	$H^2 = H_0^2 \{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\Lambda,0} - \Omega_{\text{Cass},0}(1+z)^4\}$	$H_0, \Omega_{m,0}, \Omega_{\Lambda,0}, \Omega_{\text{Cass},0}$
10 flat FRW model with Chaplygin gas and baryonic matter $p_{\text{eff}} = 0 - 3\bar{\alpha}\rho^m H$; $\Omega_{m,0} = 0.05$, $m = -1.5$ are fixed	$H^2 = H_0^2 \left\{ (1 - \Omega_{m,0}) [A_S + (1 - A_S)(1+z)^{3(\frac{1}{2}-m)}]^{\frac{2}{1-2m}} + \Omega_m(1+z)^3 \right\}$	H_0, A_S

where B is a constant and the energy density contain only dust matter and radiation ($8\pi G = c = 1$). The additional second term in relation (3) may arise from ‘new physics’. It dominates at the late epoch and drives the present acceleration of the Universe. Because terms of type ρ^n may arise as a consequence of living on $(3+1)$ brane in a high-dimensional bulk space, the origin of acceleration lies rather in modification of the FRW equation at low energy scales than due to a dark energy component. In brane world scenarios, the observer is embedded on the brane in a larger space on which gravity can propagate. The idea is that an observer measures only 4-dimensional gravity up to some corrections that given the weakness of gravity, can in general be made small enough not to conflict with observations without tweaking with a model parameter too much [19].

In Table II it is collected ten representative models offering explanation of current acceleration of the Universe in an alternative way to a dark energy. Apart from the Cardassian model there are different brane world scenarios which were originally proposed by Dvali, Gabadadze and Porrati (DGP) [20], Deffayet, Dvali and Gabadadze (DDG) [21], Randall and Sundrum [22], Shtanov brane models [23]. We also consider models of ‘nonlinear gravity’ based on modified Lagrangian density which is proportional to R^n , where R is the Ricci scalar and n is constant [24] ($n = 1$ for standard general relativity), models based on non-Riemannian gravity [25], bouncing models arising in the context of loop quantum gravity and models with energy transfer between the dark matter and dark energy sectors (Λ decaying vacuum and phantom dark energy).

The main goal of this paper is to make the ranking of accelerating models using the Bayesian framework [26, 27]. The effectiveness of application of information criteria of model selection in the cosmological context was recently demonstrated by many authors [28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44]. In the paper we obtain these accelerating models which are the best ones from the set under consideration in explaining the SNIa data. We calculate the Bayes factor for all models with different numbers of parameters, which differentiate between them [45]. The Bayes factor measures the change in relative probabilities of any two models in light of observational data (SNIa data) when we update the prior relative model probabilities to the posterior relative model probabilities.

In observational cosmology many theoretically allowed models with different prediction of the past (big bang versus bounce) and the future (big rip versus de Sitter attractor) becomes in good agreements with the observational data [46]. Therefore we propose to use the Bayesian factor to differentiate among all these models and make some ranking.

II. IDEA OF MODEL SELECTION

In the development of cosmology the basic role played an idea of cosmological models together with an idea of astronomical tests [47]. The idea of cosmological tests make cosmological models parts of astronomy which offers possibility of observationally determining the set of realistic parameters, that can characterize viable models. While we can perform estimation of model parameters using a standard minimization procedure based on the likelihood method, many different scenarios are still in a good agreement with observational data of SNIa. This problem which appears in observational cosmology is called the degeneracy problem. To solve this problem it is required to differentiate between different dark energy models. We propose to use the Akaike information criterion (AIC) and BIC quantity (as an approximation to the logarithm of the marginal likelihood).

For the model selection framework it is required to have data and a set of models and then we can make the model based statistical inference. The model selection should be based on a well-justified single (even naive) model or, at least, a simple model which suffices for making inferences from the data. In our case the Λ CDM model plays just the role of such a model. The model selection should be viewed as a way to obtain model weights, not just a way to select only one model (and then ignore that selection occurred). Moreover in the notion of true models do not believe information theories because the model by definition is only approximations to unknown physical reality: there is no true model of the Universe that perfectly reflect large structure of space-time, but some of them are useful.

In this paper the Bayes factor and AIC quantity are used to compare models gathered in Table I and Table II.

The AIC is defined in the following way [48]

$$\text{AIC} = -2 \ln \mathcal{L} + 2d, \quad (4)$$

where \mathcal{L} is the maximum likelihood and d is a number of model parameters. The best model with a parameter set providing the preferred fit to the data is that which minimizes the AIC. While there are justification of the AIC in information theory and also rigorous statistical foundation for the AIC, it can be also justified as Bayesian using a ‘savvy’ prior on models that is a function of a sample size and a number of model parameters. For the AIC we can define ΔAIC_i as the difference of the AIC for model i and the AIC value for the reference model: $\Delta_i = \text{AIC}_i - \text{AIC}_1$. The Δ_i are easy to interpret and allow a quick ‘strength of evidence’ comparison and a ranking of candidates for dark energy description. The models with $0 \leq \Delta_i \leq 2$ have substantial support (evidence), those where $4 < \Delta_i \leq 7$ have considerably less support, while models having $\Delta_i > 10$ have essentially no support with respect to model 1.

TABLE II: The Hubble function for 10 cosmological models beyond the standard general relativity

case model	$H^2(z)$ relation	independent model parameters
1 Cardassian type of Friedmann equation, $\Omega_{r,0} = 10^{-4}$ is fixed	$H^2 = H_0^2 \left\{ \Omega_{k,0}(1+z)^2 + \Omega_{m,0}(1+z)^4 \left[\frac{1}{1+z} + (1+z)^{-4+4n} \left(\frac{1-\Omega_{r,0}-\Omega_{m,0}}{\Omega_{m,0}} \right) \left(\frac{\frac{1}{1+z} + \frac{\Omega_{r,0}}{\Omega_{m,0}}}{1 + \frac{\Omega_{r,0}}{\Omega_{m,0}}} \right)^n \right] \right\}$	$H_0, \Omega_{k,0}, n$
2 Dvali-Gabadadze-Porrati brane models (DGP)	$H^2 = H_0^2 \{ [\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{rc,0}} + \sqrt{\Omega_{rc,0}}]^2 + \Omega_{k,0}(1+z)^2 \}$	$H_0, \Omega_{m,0}, \Omega_{rc,0}$
3 Deffayet-Dvali-Gabadadze brane models with λ (DDG)	$H^2 = H_0^2 \left\{ \left[-\frac{1}{2r_0 H_0} + \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\lambda,0} + \frac{1}{4r_0^2 H_0^2}} \right]^2 \right\}$	$H_0, r_0, \Omega_{\lambda,0}$
4 Randall-Sundrum brane models with dark radiation and $\Lambda = 0$	$H^2 = H_0^2 \{ \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{dr,0}(1+z)^4 + \Omega_{\lambda,0}(1+z)^6 \}$	$H_0, \Omega_{m,0}, \Omega_{dr,0}, \Omega_{\lambda,0}$
5 Randall-Sundrum brane models with dark radiation and Λ (RSB)	$H^2 = H_0^2 \{ \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{dr,0}(1+z)^4 + \Omega_{\lambda,0}(1+z)^6 + \Omega_{\Lambda,0} \}$	$H_0, \Omega_{m,0}, \Omega_{dr,0}, \Omega_{\lambda,0}, \Omega_{\Lambda,0}$
6a Shtanov brane models (Brane1)	$H^2 = H_0^2 \left\{ \Omega_{m,0}(1+z)^3 + \Omega_{\sigma,0} + 2\Omega_{l,0} - 2\sqrt{\Omega_{l,0}} \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\sigma,0} + \Omega_{l,0} + \Omega_{\Lambda b,0}} \right\}$	$H_0, \Omega_{m,0}, \Omega_{\sigma,0}, \Omega_{l,0}$
6b. Shtanov brane models (Brane2)	$H^2 = H_0^2 \left\{ \Omega_{m,0}(1+z)^3 + \Omega_{\sigma,0} + 2\Omega_{l,0} + 2\sqrt{\Omega_{l,0}} \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\sigma,0} + \Omega_{l,0} + \Omega_{\Lambda b,0}} \right\}$	$H_0, \Omega_{m,0}, \Omega_{\sigma,0}, \Omega_{l,0}$
7 modified affine gravity (MAG) model	$H^2 = H_0^2 \{ \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\psi,0}(1+z)^6 + \Omega_{\Lambda,0} \}$	$H_0, \Omega_{m,0}, \Omega_{\psi,0}, \Omega_{\Lambda,0}$
8 FRW models of nonlinear gravity with Lagrangian density proportional to Ricci scalar R (NG)	$H^2 = H_0^2 \{ \Omega_{m,0}(1+z)^3 \frac{2n}{3-n} + \Omega_{r,0}(1+z)^4 \frac{4n(2-n)}{(n-3)^2} \} \Omega_{\text{nonl},0}(1+z)^{\frac{3(1-n)}{n}}$	$H_0, \Omega_{m,0}, \Omega_{\text{nonl},0}, n$
9 bouncing models with Λ (BACDM)	$H^2 = H_0^2 \{ \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 - \Omega_{n,0}(1+z)^n + \Omega_{\Lambda,0} \}$	$H_0, \Omega_{m,0}, \Omega_{n,0}, \Omega_{\Lambda,0}$
10a models with energy transfer (dark matter \leftrightarrow vacuum energy ρ_1 sector)	$H^2 = H_0^2 \{ \Omega_{m,0}(1+z)^3 + \Omega_{\text{int}}(1+z)^n + \Omega_{\Lambda,0} \}$	$H_0, \Omega_{m,0}, \Omega_{\Lambda,0},$
10b models with energy transfer (dark matter \leftrightarrow phantom dark energy sector)	$H^2 = H_0^2 \left\{ \Omega_{m,0}(1+z)^3 + \Omega_{\text{int}}(1+z)^n + \Omega_{\text{ph}}(1+z)^{3(1+w_X)} \right\}$	$H_0, \Omega_{m,0}, \Omega_{\text{ph},0}, w_X$

In the Bayesian framework a best model (from the model set $\{M_i\}, i = 1, \dots, K$) is that which has the largest value of probability in the light of data (so called a posteriori probability). We can define the posterior odds for models M_i and M_j , which (in the case when no model is favored a priori) is reduced to the marginal likelihood (E) ratio (so called the Bayes factor – B_{ij})

$$B_{ij} = \frac{\int L(\bar{\theta}|D, M_i)P(\bar{\theta}|M_i)d\bar{\theta}}{\int L(\bar{\eta}|D, M_j)P(\bar{\eta}|M_j)d\bar{\eta}} = \frac{E_i}{E_j}, \quad (5)$$

where $\bar{\theta}$ is a parameter vector, which defines model i , $L(\bar{\theta}|D, M_i)$ is likelihood under model i , $P(\bar{\theta}|M_i)$ is the prior probability for $\bar{\theta}$ under a model i .

Schwarz [49] showed that for iid observations coming from the linear exponential family distribution the asymptotic approximation ($N \rightarrow \infty$) to the logarithm of the marginal likelihood is given by

$$\ln E = \ln \mathcal{L} - \frac{d}{2} \ln N + O(1). \quad (6)$$

According to this result he introduced a criterion for the model selection: the best model is that which minimized the BIC, defined as

$$\text{BIC} = -2 \ln \mathcal{L} + d \ln N. \quad (7)$$

This criterion can be derived in such a way that it is not required to assume any specific form for the likelihood function but it is only necessary that the likelihood function satisfies some non-restrictive regularity conditions. Moreover the data do not need to be independent and identically distributed [50].

To compare models M_i and M_j one can compute $2 \ln B_{ij} = -(\text{BIC}_i - \text{BIC}_j) \equiv -\Delta \text{BIC}_{ij}$ which can be interpret as ‘strength of evidence’ against j model: $0 \leq 2 \ln B_{ij} < 2$ – not worth more than a bare mention, $2 \leq 2 \ln B_{ij} < 6$ – positive, $6 \leq 2 \ln B_{ij} < 10$ – strong, and $2 \ln B_{ij} \geq 10$ – very strong.

It is useful to choose one model from our model set (a reference model – s) and compare the rest models with this one model, situation in which $2 \ln B_{si} > 0$ indicates evidence against model i with respect to the reference model, whereas $2 \ln B_{si} < 0$ denotes evidence in favor of model i .

We can compute posterior probability for model i in the following way

$$P(M_i|D) = \frac{B_{is}}{\sum_{k=1}^K B_{ks}}, \quad (8)$$

where $B_{is} = \exp[\frac{1}{2} \Delta \text{BIC}_{si}]$. Then one can see how prior believe about model probability $P(M_i) = \frac{1}{K}$ change after inclusion data to analysis. This is the probability for model i being the best model from set of models under consideration.

There are many simulation studies in the statistical literature on either the AIC or BIC alone, or often comparing them and making recommendation on which one to use. It should be pointed out that both of them are an asymptotic approximation. This assumption is satisfied when sample size used in analysis is large enough, large with respect to the number of unknown model parameters.

Note that the assumptions of using model selection methods are satisfied, namely

1. there is model-based inference from SNIa data (the luminosity distance observable);
2. there is a set of models and no certainty about which model should be used in explanation of present acceleration;
3. a data-based choice is made among these competing models (see Table I, Table II).

In this paper the restricted ‘Gold’ sample of $N = 157$ SNIa [1] is used. It is assumed that the supernovae measurements come with uncorrelated Gaussian errors. In this case the likelihood function has the following form

$$L \propto \exp \left[-\frac{1}{2} \left(\sum_{i=1}^N \frac{(\mu_i^{\text{theor}} - \mu_i^{\text{obs}})^2}{\sigma_i^2} \right) \right], \quad (9)$$

where σ_i is known, $\mu_i^{\text{obs}} = m_i - M$ (m_i – the apparent magnitude, M – the absolute magnitude of SNIa), $\mu_i^{\text{theor}} = 5 \log_{10} D_{Li} + \mathcal{M}$, $\mathcal{M} = -5 \log_{10} H_0 + 25$ and $D_{Li} = H_0 d_{Li}$, where d_{Li} is luminosity distance

$$d_{Li} = (1 + z_i) \frac{c}{H_0} \frac{1}{\sqrt{|\Omega_{k,0}|}} \mathcal{F} \left(H_0 \sqrt{|\Omega_{k,0}|} \int_0^{z_i} \frac{dz'}{H(z')} \right), \quad (10)$$

TABLE III: Values of the AIC, BIC, ΔAIC_{is} and $2\ln B_{si}$ (where s is the index of the reference model) for the flat models from Table I.

case	AIC	BIC	ΔAIC_{i1}	$2\ln B_{1i}$	ΔAIC_{i10}	$2\ln B_{10i}$
1	179.9	186.0	0.0	0.0	2.0	2.1
2	179.9	189.0	0.0	3.0	2.0	5.1
3	183.2	189.4	3.3	3.4	5.3	5.5
4	178.0	184.1	-1.9	-1.9	0.1	0.2
5	178.5	187.7	-1.4	1.7	0.6	3.8
6	179.7	188.9	-0.2	2.9	1.8	5.0
7	181.7	193.9	1.8	7.9	3.8	10.0
8a	180.5	192.7	0.6	6.7	2.6	8.8
8b	180.4	192.6	0.5	6.6	2.5	8.7
9	181.9	197.1	2.0	11.1	4.0	13.2
10	177.9	183.9	-2.0	-2.1	0.0	0.0

TABLE IV: Values of the AIC, BIC, ΔAIC_{is} and $2\ln B_{si}$ (where s is the index of the reference model) for the flat models from Table II.

case	AIC	BIC	ΔAIC_{i1}	$2\ln B_{1i}$	ΔAIC_{i2}	$2\ln B_{2i}$
1	178.5	187.7	0.0	0.0	-2.4	0.7
2	180.9	187.0	2.4	-0.7	0.0	0.0
3	180.8	189.9	2.3	2.2	-0.1	2.9
4	327.0	336.2	148.5	148.5	146.1	149.2
5	182.1	194.3	3.6	6.6	1.2	7.3
6a	180.3	192.5	1.8	4.8	-0.6	5.5
6b	183.8	196.0	5.3	8.3	2.9	9.0
7	180.3	189.4	1.8	1.7	-0.6	2.4
8	186.6	195.8	8.1	8.1	5.7	8.8
9	183.9	196.2	5.4	8.5	3.0	9.2
10a	180.1	192.3	1.6	4.6	-0.8	5.3
10b	180.2	192.4	1.7	4.7	-0.7	5.4

where $\Omega_{k,0} = -\frac{k}{H_0^2 a_0^2}$ and

$$\begin{aligned}
\mathcal{F}(x) &= \sinh(x) && \text{for} && k < 0 \\
\mathcal{F}(x) &= x && \text{for} && k = 0 \\
\mathcal{F}(x) &= \sin(x) && \text{for} && k > 0.
\end{aligned}$$

Table III gives the value of the AIC and BIC for flat models from Table I. It also contains values of ΔAIC_{is} and $2\ln B_{si}$, where model s is our reference model: the Λ CDM model (indexed by 1) in the first case and model which minimize both the AIC and BIC quantities – the flat FRW model with noninteracting the Chaplygin gas and baryonic matter (indexed by 10) in the second one. These values are also illustrated in Figures 1A, 1B and Figures 2A, 2B respectively. Table IV contains the same quantities for flat models from Table II. Here the first reference model is that which minimizes the AIC quantity – the Cardassian model (indexed by 1), the second one is that which minimizes the BIC quantity – the DGB model (indexed by 2). Note that in this case these models are different (on the contrary with situation in Table I, where the same model minimizes both the AIC and BIC indices). Figures 3A, 3B and Figures 4A, 4B illustrate values of ΔAIC_{is} and $2\ln B_{si}$ respectively. Additionally we compare all flat models from Table II with the Λ CDM model (those comparisons are illustrated on Figures 5A and 5B.)

In Figures 6A, 6B are presented values of $\Delta\text{AIC}_{is\text{AIC}}$ and $2\ln B_{s\text{BIC}i}$ respectively for set of brane models from Table II: $\{2, 3, 4, 5, 6a, 6b\}$. Here the reference model for the AIC analysis is the Shtanov Brane1 model (indexed by 6a), which minimizes the AIC quantity in the set under consideration, whereas the DGB model (indexed by 2) for the BIC analysis (this one minimizes the BIC quantity in the models set).

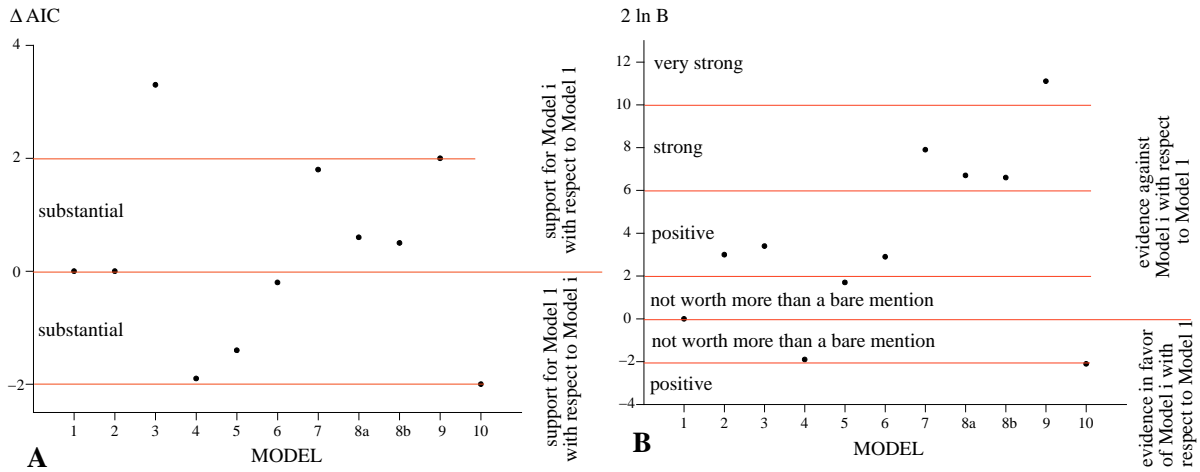


FIG. 1: Value of A) $\Delta AIC_{i1} = AIC_i - AIC_1$ and B) $2 \ln B_{1i}$ for models from Table I.

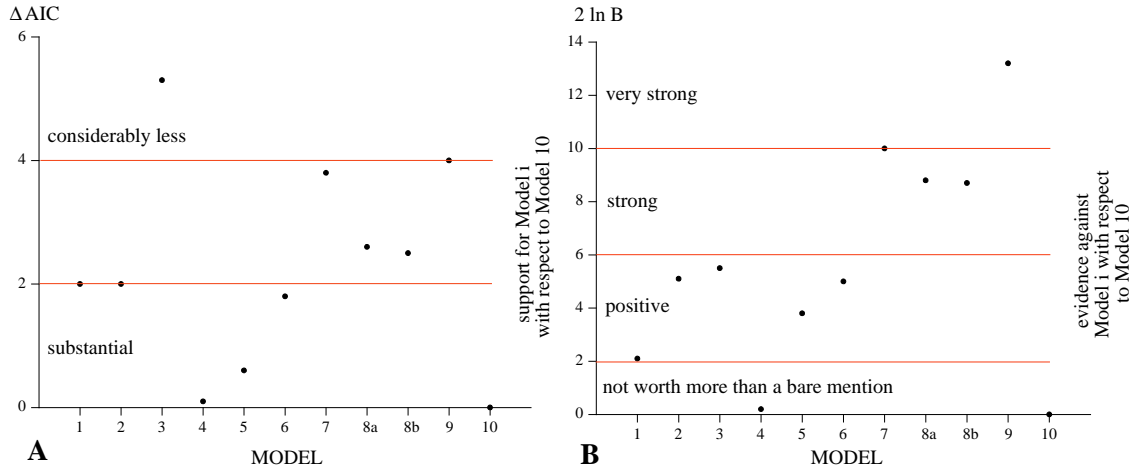


FIG. 2: Value of A) $\Delta AIC_{i10} = AIC_i - AIC_{10}$ and B) $2 \ln B_{10i}$ for models from Table I.

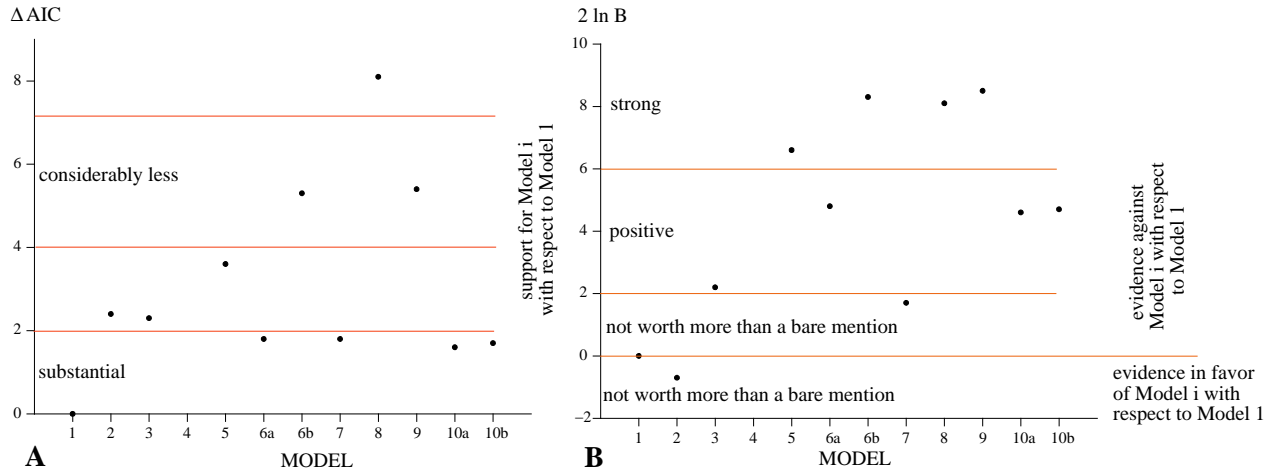


FIG. 3: Value of A) $\Delta AIC_{i1} = AIC_i - AIC_1$ and B) $2 \ln B_{1i}$ for models from Table II.

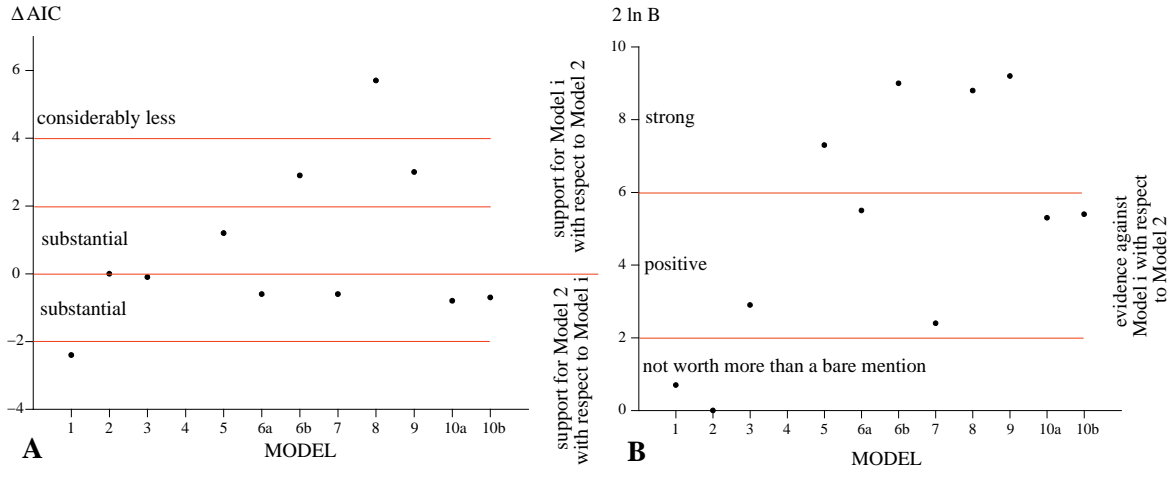


FIG. 4: Value of A) $\Delta AIC_{i2} = AIC_i - AIC_2$ and B) $2 \ln B_{2i}$ for models from Table II.

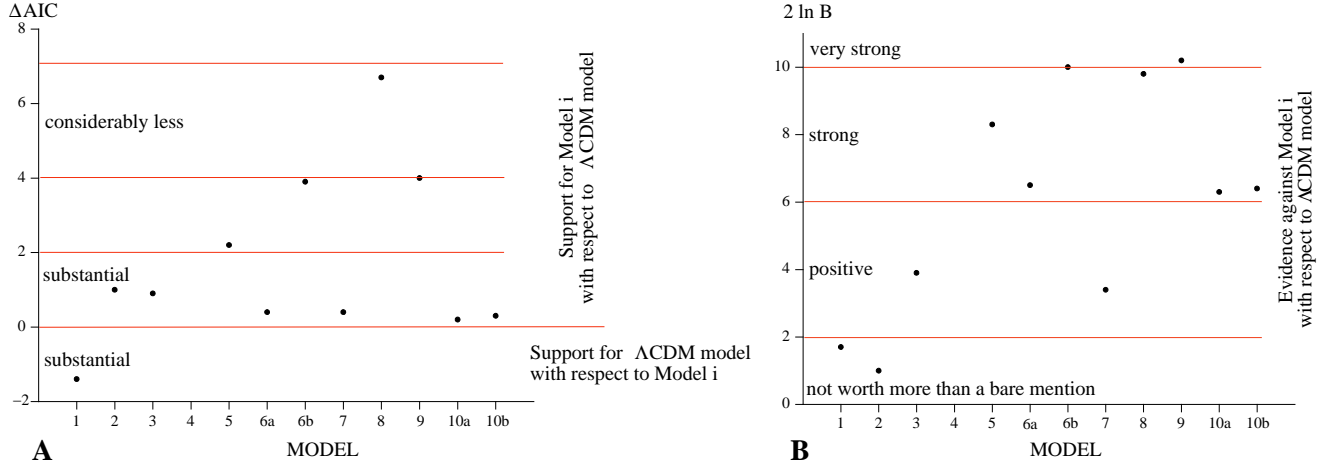


FIG. 5: Value of A) $\Delta AIC_{is} = AIC_i - AIC_s$ and B) $2 \ln B_{si}$ for models from Table II, s —the index of the reference model—the Λ CDM model.

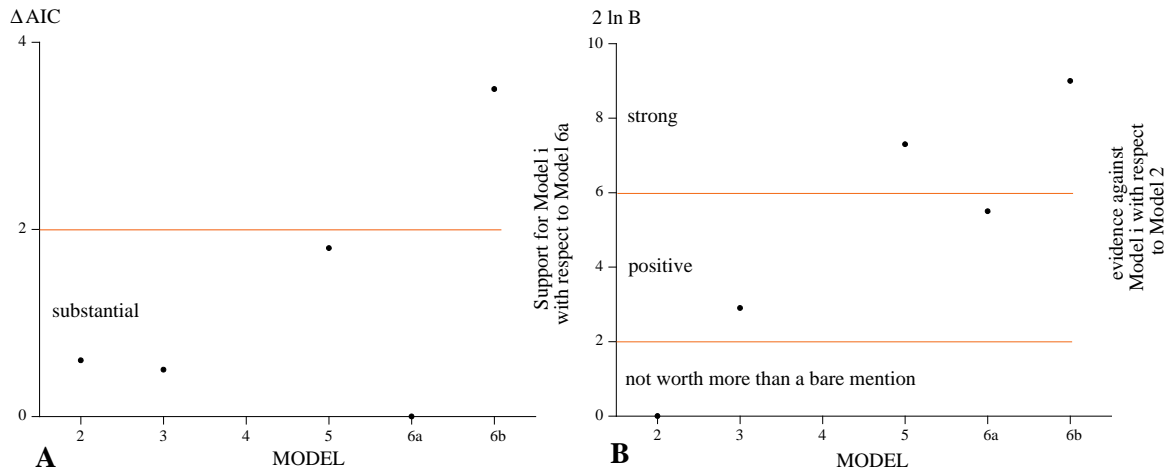


FIG. 6: Value of A) $\Delta AIC_{i6a} = AIC_i - AIC_{6a}$ and B) $2 \ln B_{2i}$ for set of brane models from Table II.

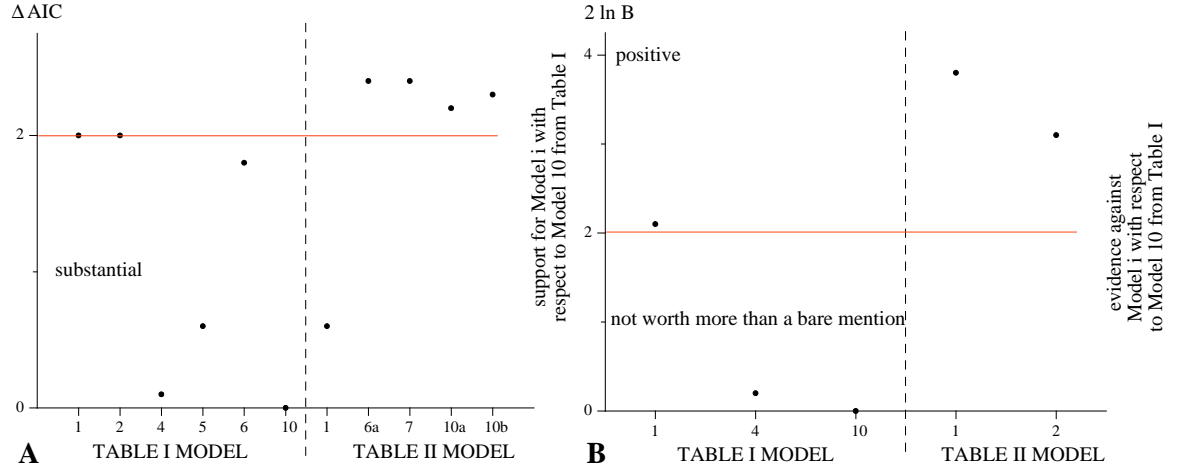


FIG. 7: Value of A) $\Delta AIC_{is} = AIC_i - AIC_s$ for the set of models from Table I: $\{1, 2, 4, 5, 6, 10\}$ and from Table II: $\{1, 6a, 7, 10a, 10b\}$, B) $2 \ln B_{si}$ for the set of models from Table I: $\{1, 4, 10\}$ and Table II: $\{1, 2\}$, s —the index of the reference model—the model 10 in both cases.

TABLE V: Values of the prior and posterior probability for models from Table I.

case	1	2	3	4	5	6	7	8a	8b	9	10
prior	0.0909	0.0909	0.0909	0.0909	0.0909	0.0909	0.0909	0.0909	0.0909	0.0909	0.0909
posterior	0.1315	0.0293	0.0240	0.3399	0.0561	0.0308	0.0025	0.0046	0.0048	0.0005	0.3757

In Figure 7A are illustrated values of ΔAIC_{is} for models which have substantial support with respect to the reference model in Figures 2A and Figure 3A: set of models from Table I: $\{1, 2, 4, 5, 6, 10\}$ together with set of models from Table II: $\{1, 6a, 7, 10a, 10b\}$. Here the reference model is this which has a minimal value of the AIC quantity in the set of models under consideration – the flat FRW model with noninteracting Chaplygin gas and baryonic matter (model indexed by 10 from Table I). In Figure 7B are presented values of $2 \ln B_{si}$ for models from Figures 2B and 4B for which evidence against them with respect to the reference model is not worth more than a bare mention: Table I: $\{1, 4, 10\}$ and Table II: $\{1, 2\}$. Here the reference model is this which minimizes the BIC quantity – the model indexed by 10 from Table I.

In Tables V, VI, VII and VIII are gathered values of prior and posterior probabilities (see (8)) for models from Table I, Table II, set of brane models from Table II, set of the best models from Table I and Table II respectively.

III. CONCLUSION

Main aim of the paper was exploring the Bayesian framework of model selection into discussion which cosmological model describe present accelerating phase expansion of the Universe. In principle there are two types of explanation why current Universe is accelerating. In the first group it is postulated existence of perfect fluid which violate the strong energy condition. The nature (Lagrangian) of such matter called dark energy is unknown although the cosmological constant is the most popular candidate for dark energy description.

The second group of explanations is based on hypothesis that dark energy could actually be the manifestation of a modification to the Friedmann equation arising from ‘new physics’ (e.g. extra dimensions, generalized general relativity, etc.). Calculating the corrections to the standard FRW equation we can explore the phenomenology and different evolutionary scenarios. Therefore there is no single hypothesis, instead there are several well-supported hypotheses (i.e. dark energy models) that are being entertained. It is just realization of concept of ‘Multiple Working

TABLE VI: Values of the prior and posterior probability for models from Table II.

case	1	2	3	4	5	6a	6b	7	8	9	10a	10b
prior	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083
posterior	0.122	0.173	0.041	0.000	0.004	0.011	0.002	0.052	0.285	0.285	0.012	0.012

TABLE VII: Values of the prior and posterior probability for set of brane models from Table II.

case	2	3	4	5	6a	6b
prior	0.167	0.167	0.167	0.167	0.167	0.167
posterior	0.749	0.175	0.019	0.000	0.047	0.008

TABLE VIII: Values of the prior and posterior probability for set of models from Table I: $\{1, 4, 10\}$ and Table II: $\{1, 2\}$.

case	1	4	10	1	2
prior	0.20	0.20	0.20	0.20	0.20
posterior	0.13	0.34	0.38	0.06	0.08

Hypotheses' advocated by Chamberlin [51]. In Tables I and II we completed the 20 models in two classes which cover both types of explanation of acceleration of the Universe. Then we adopt the model selection methods to obtain 'strength of evidence' comparison and ranking of candidate hypotheses of dark energy. Providing quantitative information to judge the 'strength of evidence' is a key point of ranking models. The hypothesis testing (which only provides qualitative information significant vs. nonsignificant) is particularly limited in the model selection (for discussion of the likelihood based strength of evidence see Ref. [52]).

Hence we obtain the set of models which are recommended by the AIC and Bayes factor. The results are following:

1. While the AIC recommended dark energy models which belong to the set $\{10, 4, 5, 6, 1, 2\}$ the Bayes factor is more restrictive because recommended models which are elements of subset $\{10, 4, 1\}$. This is due to different values of the coefficient in the definition of AIC and BIC quantities, so called penalty term for more complex models. In the AIC definition this coefficient is always equal 2 whereas in the BIC definition it depends on simple size (here it is nearly equal 5).
2. Analogous recommendation can be performed for the class of models with a modified Friedmann equation, namely the AIC recommends a set of models $\{1, 8, 10a, 10b, 6a, 7\}$ and the Bayes factor favors class of models $\{2, 1\}$ which has non-empty intersection with models recommended by the AIC.
3. One can construct also ranking within the best models in each category I and II by AIC and Bayes factor respectively. Then we obtain a set of models recommended by the AIC $\{10, 4, 5, 6, 1, 2\}_I \cup \{1\}_{II}$ and by the Bayes factor $\{10, 4, 1\}_I$.

Note that both quantities pointed out that model with noninteracting Chaplygin gas and baryonic matter is the best one from both sets of models under consideration. This is due to more a priori information about this model which we include in calculation. Two model parameters were fixed here: $\Omega_{m,0}$ and m . In situation when both parameters are fitted the conclusion is changed. All additional information which we include in calculation are necessary and can change our final inference.

It should be point out that dark energy models better explain SNIa data than models with a modified Friedmann equation.

4. We can perform ranking of the models within 'brane paradigm' and then we obtain models $\{6a, 3, 2, 5\}$ recommended by the AIC and model $\{2\}$ preferred by the Bayes factor.
5. One can conclude that the flat Λ CDM model (model indexed by 1) has still substantial support with respect to better models. Note that the AIC indicates that there is no difference between the Λ CDM ($k = 0$) model and the Λ CDM ($k \neq 0$) model (indexed by 2), both of them fit the data equally well. Whereas the Bayes factor denotes that there is an evidence against model 2 with respect to model 1.

Note that Bayes factor indicates that the flat Λ CDM model better explains SNIa data than models gathered in Table II.

6. For completeness we calculate posterior probability which measures probability for the model being the best one among the class of models under consideration (being the most favored model by data in hand). Then one can observe how prior believe about model probability change after inclusion data to analysis (see Tables V, VI, VII, VIII). Note that for all models recommended by the BIC posterior probability is greater than the prior one, whereas for models which do not belong to a preferred set of models the posterior is smaller than the prior.

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